Fully 4D Cardiac Gated SPECT Reconstruction with Simultaneous Compensation in KL Domain

Yi Fan, Chun Jiao, Hongbing Lu, Su Wang and Zhengrong Liang

Abstract — **Recent researches have shown promise in applying KL transform to 4D gated sinogram for pre-reconstruction temporal smoothing and quasi-4D inversion of attenuated Radon transform. To achieve quantitative 4D reconstruction, this work aims to compensate for the major degradation factors, including distance-dependent collimator resolution variation and object-specific photon scatter, simultaneously within the KL framework. To alleviate the influence of cardiac motion on reconstruction, heart motion was classified into several groups based on inter-frame similarities and each group underwent a corresponding KL transform. In the KL domain, non-stationary Poisson noise was stabilized by Anscombe transform and treated by adaptive Wiener filtration. Scatter contribution to the primary energy window was then estimated and removed based on photon detection energy spectrum and the triple-energy-window acquisition formula after noise treatment. The scatter-corrected data was further subject to a depth-dependent deconvolution, based on the distance frequency relationship, with measured detector response kernel in the KL domain. The deconvoluted sinograms were reconstructed by inverting the attenuated Radon transform for each KL component and the 4D SPECT images were obtained by a corresponding inverse KL transform for each group. The simultaneous compensation strategy in the KL domain was tested by computer simulations from digital phantoms of 128 cubic array and clinical data from a patient. The adaptive KL transform for different groups consisting of frames with similar activity dynamics showed noticeable improvement over our previous work of using a single KL transform for all frames. Improvement was also seen by the adaptive noise treatment of all the KL components over previous work of discarding the higher-order components. Further improvement by considering the scatter and resolution variation was demonstrated.**

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***Index Terms* — Image reconstruction, non-uniform attenuation, noise reduction, photon scatter, detector response**

I. Introduction

In single photon emission computed tomography (SPECT), Poisson noise distribution, non-uniform attenuation, photon scatter as well as detector response are the dominating degradation factors which suffer the image quality. To achieve quantitative reconstruction for clinical application, those factors should be compensated carefully. While, for fully 4D reconstruction, it is difficult to seek for an analytical solution which could consider multi-degradation factors simultaneously. A lot of efforts have been devoted to include a penalty for a penalized maximum likelihood (pML) solution, where the intra- and inter-frame correlations are considered in the penalty [1]. This classic approach is attractive because it searches for a statistical optimal solution but has several drawbacks, e.g., the reconstruction is time consuming because the solution is numerically tractable only by iterative algorithms and furthermore the solution strongly depends on several freely-adjustable parameters in the penalty. An alternative approach has been explored by the use of the Karhune-Loève (KL) transform to address the temporal correlation [2]. Our previously works showed that the proposed approach could compensate for non-uniform attenuation together with the treatment of Poisson noise distribution in KL domain accurately [3-4].

In this paper, we extend our previously work to a thoroughly frame work by incorporating well studied algorithms aim to compensate the degradation factors discussed above. Based on the linear property of KL transform, we proved that the system matrix of SPECT remain same both in temporal space and in KL domain. So, the well studied FBP-Type algorithm could be applied directly in KL domain to compensate for the non-uniform attenuation exactly and reconstruct the de-correlated principle components [5-7]. Also, we proved that the Poisson distribution property remained in KL domain, thus our well designed Wiener filter could smooth the noise very well in KL domain [8]. The contribution of photon scatter is treated simply by the triple-energy-window acquisition method [9] and the depth-dependent deconvolution is adopted to correct the detector response separately in KL space [10]. Considering the properties of degradation factors and validated by our simulation study, the following flowchart of compensation scheme in KL domain were adopted in this study.

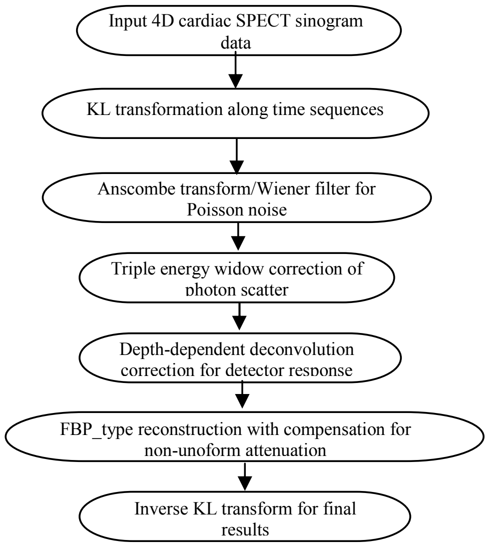


Fig.1: Flowchart of proposed KL domain multi-degradation factors compensation scheme

This paper is organized as follows: the flowchart of proposed KL domain compensation/correction scheme is introduced in section II. The system model in KL domain together with the temporal KL transformation of gated SPECT data sequence is described in section III. The application of well studied compensation algorithms in KL domain for degradation factors are shown in section IV. The simulation study and the conclusion on this approach are presented in section V and section VI respectively.

II. System Model

The acquired dynamic projection data sequence y from gated cardiac SPECT can be expressed mathematically as the attenuated Radon transform of the source distribution function λ, which reflects the mean number of gamma photons emitted by a radiotracer injected into the patient's body. The dynamic imaging procedure can be simply modeled as [7]:

$$E[y] = H\lambda \tag{1}$$

where $y = [y_1^T, y_2^T, \cdots y_K^T]^T$, $\lambda = [\lambda_1^T, \lambda_2^T, \cdots \lambda_K^T]^T$, and T denotes the transpose operation. Notation y_k (k=1, 2,…, K, where K is the total number of time frames sampled within the cardiac cycle) represents a $L\times1$ vector (L is the number of sinogram data points for each frame) obtained by lexicographically ordering the sinogram data, and λ_k represents a $N\times1$ vector (N is the number of pixels of the source image for each frame). $H = diag[H_1, H_1, \cdots, H_1]$, where H_1 is the system matrix that applies to each single frame of data, representing the spatial-temporal system matrix of size $LK \times NK$.

Assume the measured time-activity curve at pixel (i,j) of gated cardiac SPECT is represented by:

$$\lambda_{i,j}^{time} = [\lambda_{i,j}^1, \lambda_{i,j}^2, \cdots, \lambda_{i,j}^K]^T \tag{2}$$

Element (k,l) of the time covariance matrix P^{time} can be estimated from all the gated frames by:

$$[P^{time}]_{k,l} = \frac{1}{N-1}\sum_{i,j}(\lambda_{i,j}^k - \bar{\lambda}^k)(\lambda_{i,j}^l - \bar{\lambda}^l) \tag{3}$$

where $N=I*J$ is the total number of pixels in one frame and $\bar{\lambda}^k$ represents the estimated mean value of frame k by:

$$\bar{\lambda}^k = \frac{1}{N}\sum_{i,j}\lambda_{i,j}^k \tag{4}$$

By singular value decomposition, the eigenvectors M of P^{time} can be obtained from:

$$P^{time} M^T = M^T \cdot V \tag{5}$$

where $V = diag\{v_1, v_2, \cdots, v_k, \cdots, v_K\}$ and v_k denotes the k-th eigenvalue of P^{time}.

By multiplying the time-activity of pixels one-by-one with M, the temporal KL transform of the gated data can be performed by:

$$A = M \cdot \lambda_{i,j}^{time} \tag{6}$$

where $A_{i,j} = [a_{i,j}^1, a_{i,j}^2, \cdots, a_{i,j}^K]^T$ and $a_{i,j}^k$ is the k-th KL domain element at pixel (i,j). The KL transformed sequence then can be obtained by organizing $A_{i,j}$ for the corresponding pixels (i,j) in the KL domain. Since the KL transform is applied along the time dimension and the mean of each frame is computed from all N pixels in that frame using equation (4), all these N pixels are subjected to the same transformation.

To express the temporal KL transform of the entire dynamic sequence in a matrix format, we define M_L by:

$$M_L = M \otimes I_L \tag{7}$$

where I_L is the $L\times L$ identity matrix and $\otimes$ is the Kronecker product. By multiplying M_L to both sides of the system model (1) and following the same schedule given by [7], we have:

$$\begin{aligned} M_L E[y] &= M_L H\lambda = (M \otimes I_L)(I_K \otimes H_1)\lambda \\ &= (MI_K) \otimes (I_L H_1)\lambda = (I_K M) \otimes (H_1 I_N)\lambda \\ &= (I_K \otimes H_1)(M \otimes I_N)\lambda = HM_N\lambda. \end{aligned} \tag{8}$$

If we define KL transformed data with:

$$\tilde{\lambda} = M_N\lambda, \quad and \quad \tilde{y} = M_L y \tag{9}$$

The relationship between transformed gated projection data and transformed image sequence can be reflected by:

$$E[\tilde{y}] = H\tilde{\lambda} \tag{10}$$

Please note that (10) has the same form as (1), which indicates that the KL domain model is the same as that in the original (or spatio-temporal) domain. This means that the system matrix H is exactly the same in both situations, and thus any well studied reconstruction algorithms could be applied directly in KL domain without modification.

III. Noise Model

It is well recognized that the noise propagation obey Poisson distribution in SPCET sinogram. For the purpose of study the noise property in KL domain, we simplify the problem by applying the KL transform to those "pure noise" pixels along time sequences, as follows:

By expanding the KL transformation matrix M_L, and apply the KL transform to noise signals N, which is also organized as time sequences along time axis, we got:

$$M_L \times N = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,l} \\ m_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ m_{l,1} & \cdots & \cdots & m_{l,l} \end{bmatrix} \times \begin{bmatrix} n_1 \\ n_2 \\ \cdots \\ n_l \end{bmatrix} = \sum_{i=1}^{l}\sum_{j=1}^{l} m_{i,j} n_j = \begin{bmatrix} \sum_{j=1}^{l} m_{1,j} n_j \\ \sum_{j=1}^{l} m_{2,j} n_j \\ \cdots \\ \sum_{j=1}^{l} m_{l,j} n_j \end{bmatrix} \quad (11)$$

Each element in the right side of (11) represents the obtained noise signal after KL transform. A simple deduction of the transformed signals still follow the Poisson distribution is described as appendix.

IV. MULTI-COMPENSATION METHOD IN KL DOMAIN

Based on our previous work, our KL domain 2D smoothing strategy could be expressed firstly by applying the Anscombe transform to all the principle components, which converts Poisson distributed noise into Gaussian distributed one with a constant variance. That is, if x is Poisson distributed with mean equal to λ, then $y = (x + 3/8)^{1/2}$ can be approximated as Gaussian distributed with mean equal to $(\lambda + 1/8)^{1/2}$ and variance of 0.25. Therefore, the noise becomes signal independent and can be expressed mathematically as an additive term. Then our well designed Wiener filter, expressed by (12), can be used to smooth the noise accurately:

$$H(\omega_s, k_\theta) = \frac{S_{pp}(\omega_s, k_\theta)}{S_{xx}(\omega_s, k_\theta)} = \frac{S_{xx}(\omega_s, k_\theta) - S_{nn}(\omega_s, k_\theta)}{S_{xx}(\omega_s, k_\theta)} \quad (12)$$

where $S_{xx}(\omega_s, k_\theta), S_{pp}(\omega_s, k_\theta)$ and $S_{nn}(\omega_s, k_\theta)$ represents the power of acquired data, ideal data and noise data respectively.

After suppression of Poisson noise, the scatter contribution to the primary energy window measurements is then removed by our scatter estimation method, which is based on the detection energy spectrum and modified from the triple-energy-window acquisition protocol. The scattered photon counts within the main window can be described as the following equation:

$$C_s = \frac{2}{3} \times \left(\frac{C_l}{w_l} - x \cdot \frac{C_h}{w_h} \right) \times w \quad (13)$$

where w_l, w_h, w are the width of lower, higher and center (main) energy windows, C_l, C_h are the detector photons from lower and higher energy window, C_s is the estimated photon from main energy window. x is the adjusted parameter determined by measuring a point source in air.

For correction of detector response, the inversion methods proposed up to date can provide either approximate solutions that realistically characterize the resolution kernel in a real SPECT system, or exact solutions that approximate the resolution kernel to some special functional forms in order to satisfy the mathematical derivations. Our previous study showed that an accurate consideration of the measured resolution kernel is needed to demonstrate robust performance and artifact-free reconstruction and, therefore, is a better choice for quantitative SPECT imaging. In this study, the resolution variation is corrected by the depth-dependent deconvolution, which, being based on the central-ray approximation and the distance-frequency relation, deconvolves the scatter-corrected data with the accurate detector-response kernel in frequency domain.

Let $P(l, \omega_t)$ and $\widetilde{P}(l, \omega_t)$ be the 2D Fourier transform (FT) of the sinogram $\{p_i\}$ and the deblurred sinogram $\{\widetilde{p}_i\}$ respectively, where l is the angular frequency and ω_t is the spatial frequency, the distance-frequency relation is expressed as:

$$P(l, \omega_t) = H(-l/\omega_t, \omega_t)\widetilde{P}(l, \omega_t) \quad (14)$$

Where $H(d, \omega_t)$ is the 1D FT of the 2D detector-response kernel h at depth d. The deconvolution is performed in frequency domain by:

$$\widetilde{P}(l, \omega_t) = H^{-1}(-l/\omega_t, \omega_t)P(l, \omega_t) \quad (15)$$

Non-uniform attenuation compensation could be achieved through FBP-type reconstruction based on Novikov's explicit inversion formula with realistic human thoracic attenuation map.

Let (x,y) be the stationary coordinate in the image domain and (t, θ) be the rotation coordinate in the sinogram space. As shown in the last paragraph of Section II, the Novikov's formula could be used directly in the KL-domain. Following the analysis in [5-6, 12], the KL domain Novikov's inverse formula can be expressed as:

$$\phi(\vec{r}) = \frac{1}{4\pi} div \int_0^{2\pi} \vec{j}[\exp([D\mu]_\theta(s,t))\widetilde{q}(t,\theta)]\Big|_{\substack{s=\vec{r}\bullet\vec{j} \\ t=\vec{r}\bullet\vec{k}}} d\theta \quad (16)$$

where $\vec{j} = (\cos\theta, \sin\theta)$, $\vec{k} = (-\sin\theta, \cos\theta)$, ***div*** is the divergence operation, $\phi(\vec{r})$ is the reconstructed image frame from its corresponding sinogram data frame $A(t, \theta)$ in the KL domain and

$$\widetilde{q}(t,\theta) = e^{-h_1}\{\cos(h_2)\widetilde{q}_1(t,\theta) + \sin(h_2)\widetilde{q}_2(t,\theta)\} \quad (17)$$

$$\widetilde{q}_1(t,\theta) = \hat{H}\cos(h_2)e^{h_1}A(t,\theta) \quad (18)$$

$$\widetilde{q}_2(t,\theta) = \hat{H}\sin(h_2)e^{h_1}A(t,\theta) \quad (19)$$

with $h_1 = ½[R\mu](t,\theta)$, $h_2 = [\hat{H}h_1](t,\theta)$. The operators $\hat{H}$, D, and R represent the Hilbert transform, the divergent beam transform, and the Radon transform, respectively, and are defined as follows:

$$[\hat{H}g](s) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{g(\tau)}{s-\tau}d\tau \qquad (20)$$

$$[D\mu]_{\theta}(s,t) = \int_{t}^{\infty}\mu_{\theta}(s,\tau)d\tau \qquad (21)$$

$$[R\mu](s,\theta) = \int_{-\infty}^{\infty}\mu_{\theta}(s,t)dt \qquad (22)$$

The above quantitative reconstruction is performed frame-by-frame in the KL domain for each principal component, which is similar to that performed in the spatial domain, and the result is $\Phi_{m,n} = (\phi^{1}_{m,n}, \phi^{2}_{m,n}, \cdots \phi^{l}_{m,n}, \phi^{l+1}_{m,n}, \cdots, \phi^{K}_{m,n})$ for each image pixel (*m*,*n*) in the KL domain. Since the higher-order components with smaller eigenvalues may have little information, only the resulted frames reconstructed from the first *l* low-order components, i.e. $\Phi^{l}_{m,n} = (\phi^{1}_{m,n}, \phi^{2}_{m,n}, \cdots \phi^{l}_{m,n})$ ($l \le K$), could be retained for further noise reduction and computing efficiency. After Novikov's inversion in the KL domain, an inverse KL transform on the *K* or *l* reconstructed frames will generate the gated images in the original space of

$$\hat{\lambda}^{time}_{m,n} = M^{T}\Phi \qquad (23)$$

V. Experiment Results

Computer simulations were conducted to show the feasibility of the presented analytical reconstruction for quantitative gated SPECT and its potential in practical use. Phantom study was based on the 3D chest phantom with Monte-Carlo simulation of the photon scatter in three energy windows. The torso attenuation map was specified by the given organs and their locations inside the body, as shown in Fig. 2. The phantoms are of 128 cubic sizes and the sinogram were collected by 128 detector bins with 128 angular samples evenly spaced on 360 degree. The depth-dependant curves for the collector bins are given in Fig. 3.

To test the performance of proposed method on single/multi-degradation factors, groups of correction strategies were adopted for comparison purpose. Figure 4, 5 and 6 show the results obtained through our method and those without compensation.

(a) (b)

Fig.2: Selected image from 8 gated chest phantom (a) and attenuation map (b).

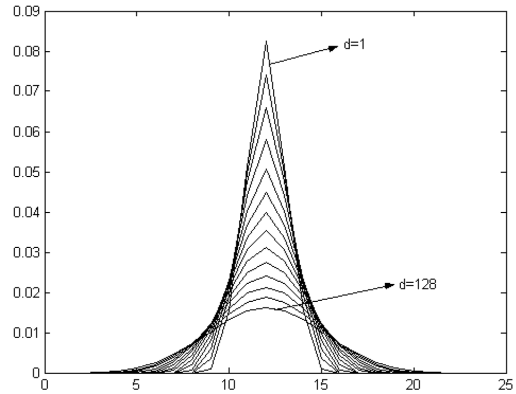
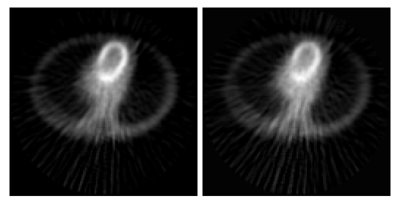
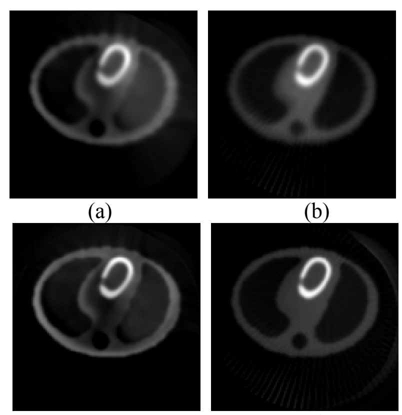


Fig.3: Point spread curve.



(a) (b)

Fig.4: Comparison of proposed scheme over photon scatter, (a) reconstruction without correction of photon scatter and (b) reconstruction with correction of photon scatter.



(a) (b)

(c) (d)

Fig.5 Comparison of proposed scheme over non-uniform attenuation and detector response, (a) reconstruction without correction of non-uniform attenuation and detector response, (b) reconstruction with correction of non-uniform attenuation but without detector response. (c) reconstruction with correction of detector response but without non-uniform attenuation, (b) reconstruction with correction of both non-uniform attenuation and detector response.

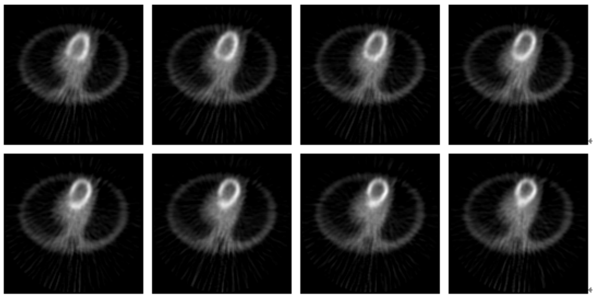


Fig.6: reconstruction with compensation of all degradation factors by proposed method for 8 gated chest phantom.

VI. DISCUSS AND CONCLUSION

In this paper, an analytical reconstruction scheme for quantitative gated SPECT was presented. The temporal correlation among the gated sequence of sinogram frames was considered by the KL transform. In the KL domain, well studied algorithms were used step by step aim to improve the image quality by correcting the degradation factors carefully. Quantitative inversion could be achieved through inverse KL transform.

The presented analytical reconstruction of gated SPECT has several advantages over the conventional FBP method such as (1) having quantitative capability by compensating for the non-uniform attenuation and (2) being more effective in noise reduction because of the unique SNR distribution among the KL principal components, where the noise treatment can be adaptive to each component. Photon scatter and detector response were considered in this frame work for further image quality improvement.

The presented analytical inversion approach also has several. In the presented KL-transform along the time sequence, time covariance matrix was computed using all pixels in each frame. In other words, all the pixels in each frame underwent the same transformation. This may add in more edge blurring in the inversion results. This drawback could be mitigated by clustering the pixels into subgroups and applying spatially-variant KL transform on each subgroup, as proposed in [13]. This adaptive KL transform strategy and other possible solutions are currently under investigation [14]. And also, the step by step compensation strategy in KL domain through analytical approach will inevitably incorporate errors due to algorithms itself or the computational implementation and will gradually accumulated thus might lead to a sharp input error for final results.

APPENDIX

To validate the noise distribution property in KL domain, we decompose the problem described in section III as two parts. Each element in formula (11) could be deemed as a linear transform of Poisson distribution n_i which has the mean value λ_i. Then, for any group of two independent variables $m = n_i + n_j$, we can obtain:

$$
\begin{aligned}
P(m=i) &= \sum_{k=0}^{i} \frac{\lambda_i^k \lambda_j^{i-k}}{k!(i-k)!} e^{-(\lambda_i+\lambda_j)} \\
&= \frac{e^{-(\lambda_i+\lambda_j)}}{i!} \sum_{k=0}^{i} \frac{i!}{k!(i-k)!} \lambda_i^k \lambda_j^{i-k} \\
&= \frac{e^{-(\lambda_i+\lambda_j)}}{i!} \sum_{k=0}^{i} \binom{i}{k} \lambda_i^k \lambda_j^{i-k} \\
&= \frac{e^{-(\lambda_i+\lambda_j)}}{i!} \left(\lambda_i+\lambda_j\right)^i \\
&= \frac{\left(\lambda_i+\lambda_j\right)^i e^{-(\lambda_i+\lambda_j)}}{i!}, \qquad i = 0,1,2,\cdots
\end{aligned}
$$

It denotes that m is also Poisson distributed with the mean value $\lambda_i + \lambda_j$. This conclusion could be used to formula (11) directly. It is easily to prove that each element should obey Poisson distribution with mean value equal to $\sum_{j=1}^{I} m_{i,j}\lambda_j$.

REFERENCES

[1]. D.S. Lalush and B.M.W. Tsui, "Block iterative techniques for fast 4D reconstruction using *a priori* motion models in gated SPECT", *Phys. Med. Biology*, vol. 43, pp. 875-886, 1998.

[2]. M.V. Narayanan, M.A. King, E.J. Soares, C.L. Byrne, P.H. Pretorius, and M.N. Wernick, "Application of the Karhunen-Loeve transform to 4D reconstruction of cardiac gated SPECT images", *IEEE Trans. Nucl. Science*, vol. 46, pp. 1001-1008, 1999.

[3]. Y. Fan, H. Lu, C. Hao, and Z. Liang. "Analytical Reconstruction of 4-D Dynamic Cardiac SPECT with Noise-Reduction and Non-Uniform Attenuation Compensation", *Nuclear Instruments and Methods in Physics Research A*, Vol. 571, 195-198, 2007.

[4]. Y. Fan, H. Lu, C. Hao, Z. Liang, and Z. Zhou. "Fast Analytical Reconstruction of Gated Cardiac SPECT with Non-Uniform Attenuation Compensation", *International Journal of Image and Graphics,* Vol. 7, 87-104, 2007.

[5]. R.G. Novikov, "An inversion formula for the attenuated X-ray transformation", *Comptes Rendus de l'Académie des Science* (Series I-Mathematics), vol. 332(12), pp. 1059-1063, 2001, and in *Ark. Math.*, vol. 40, pp. 145-167, 2002.

[6]. F. Natterer, "Inversion of the attenuated Radon transform", *Inverse Problems*, vol. 17, pp. 113-119, 2001.

[7]. J. You, G.L. Zeng, and Z. Liang, "FBP algorithms for attenuated fan-beam projections", *Inverse Problems*, vol. 21, pp. 1179-1192, 2005.

[8]. T. Li, J. You, J. Wen, and Z. Liang, "An efficient reconstruction method for non-uniform attenuation compensation in non-parallel beam geometries based on Novikov's explicit inversion formula", *IEEE Trans. Med. Imaging*, vol. 24, pp. 1357-1368, 2005.

[9]. H. Lu, G. Han, D. Chen, L. Li, and Z. Liang, "A theoretically based pre-reconstructing filter for spatio-temporal noise reduction in gated cardiac SPECT imaging", *Conference Record IEEE NSS-MIC*, in CD-ROM, 2000.

[10]. X Li, G. Han, H Lu, L Li, and Z Liang, "A new scatter estimation method using triple window acquisition to fit energy spectrum", *J. Nucl. Med.* 42, pp. 194, 2001.

[11]. R.M. lewitt, P.R.Edholm, and W.Xia, "Fourier method for correction of depth dependent blurring," *SPIE Medical Imaging III*,vol. 1092, 232-239,1989.

[12]. L.A. Kunyansky, "A new SPECT reconstruction algorithm based on the Novikov's explicit inversion formula", *Inverse Problems*, vol. 17, pp. 293-306, 2001.

[13]. J. G. Brankov, Y. Yang, M. V. Narayanan, and M. N. Wernick, "Spatially adaptive temporal smoothing for reconstruction of dynamic PET and gated SPECT images," *Conference Record of the 2000 IEEE Nucl.. Sci. Symp. Med. Imaging Conf.*, vol. 2, pp. 146-150, 2000.

[14]. Y. Fan, H. Lu C. Hao, Z. Liang. "Improved KL Domain Inversion of

$Z_{i1}^{(n+1)} = X_{i}^{(n)}(\sigma$

ne Attenuated Radon Transform for Quantitative Gated Cardiac PECT". *Journal of Nuclear Medicine*. Vol. 48, 421, 2007.