

A Noise Reduction Method for Non-Stationary Noise Model of SPECT Sinogram based on Kalman Filter

Xiang Li, Hongbing Lu, Guoping Han and Zhengrong Liang

Department of Radiology, State University of New York, Stony Brook, NY 11794, USA

Abstract: The non-stationary Poisson noise in single photon emission computed tomography (SPECT) sinogram is a major cause to compromise the quality of reconstructed images and challenge any compensation strategy for photon attenuation, scatter and collimator response. Research utilizing three-dimensional Wiener filter after Anscombe transformation, which converts Poisson distributed noise into Gaussian distributed one, has been conducted recently with noticeable success. However, the prerequisite of stationary random process for the Wiener approach is not exactly valid for the SPECT sinogram. In this paper, the non-stationary Poisson noise was first modulated by the Anscombe transformation and then the modulated noise model was analyzed. A Kalman filter for the modulated non-stationary noise model was designed to extract the means or signals from the Poisson noise sinogram. Monte Carlo program was used to generate projection data from the MCAT phantom, simulating SPECT data acquisition. The reconstructed results demonstrated a significant improvement with the Kalman filter, as compared to the Wiener approach. Dramatic improvement is seen, as compared to linear low-pass filters, at noisy levels of 100 thousand counts in a 128x128x128 sinogram size. The capability of Kalman filter for non-stationary noise model was theoretically proved and experimentally demonstrated.

I. INTRODUCTION

Poisson noise in SPECT (single photon emission computed tomography) projection data or sinogram dramatically reduces the quality of the reconstructed images. Increasing the radiotracer concentration or sampling time can improve the signal-to-noise ratio (SNR), but results in more radiation dose or motion artifact. An alternative approach is to design appropriate filter to suppress the noise.

However, the conventional linear low-pass filters cannot obtain good results for the non-stationary Poisson noise. A popular and effective method is to apply the Anscombe transformation [1], which converts the Poisson noise into Gaussian noise with a nearly constant variance of 0.25. The transformation, although not completely eliminate the non-stationary effect, enables a Wiener filter to effectively treat the noise, in addition to other iterative techniques [2][3][4]. Although Wiener filter has achieved a noticeable success, its strong dependence on the stationary random process, which means that the variance of the noise must be constant, limits further improvement. This limitation is especially significant for cases of very low count densities, as seen in most SPECT imaging studies. Therefore, further research beyond the Wiener filter approach is needed.

Kalman filter has been widely used in processing of video images [5][6]. A fundamental property of Kalman filter is its ability to trace the non-stationary variation within an image. It requires prior information of the noise model. However, prior research on Kalman filter has not succeeded for Poisson noise in low count cases. This may be due to the wide range of the Poisson noise variance.

The objective of this paper is to present a new noise reduction method, which designs the Kalman filter for the Poisson noise model after the variance is significantly reduced by the Anscombe transformation.

II. METHOD

Anscombe Transformation

It is known that the Poisson noise in SPECT sinogram is signal-dependent, which means that the variance of the Poisson noise depends on its mean value. Most of the reported noise reduction methods depend on the stationary Gaussian distribution. In mathematics, the Anscombe transformation has been proved to transform the Poisson distribution into the Gaussian distribution. If x is Poisson distributed with mean equal to λ , then $y = (x+3/8)^{1/2}$ can be approximated as Gaussian distributed with its mean equal to $(\lambda+1/8)^{1/2}$ and its variance approximately equal to 0.25 [1][2].

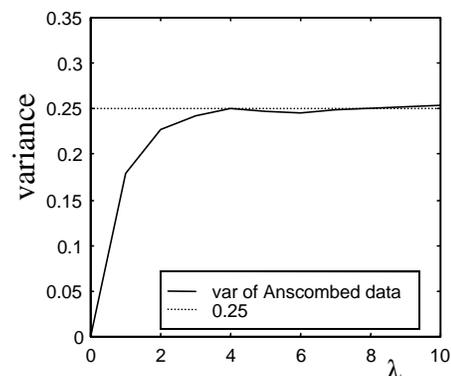


Figure 1. Variance of the Anscombe transformed data.

Figure 1 shows the relationship between the variance after Anscombe transformation and the mean λ of the original signal. It can be seen that when the λ value is larger than 4, the variance of the Anscombe transformed data with Poisson noise is approximately equal to 0.25. However, when the λ value is less than 4, the variance sharply decreased. Although

the stationary property of Anscombe transformed projection data is obviously better than that of the original sinogram with Poisson noise, it is not exactly stationary yet. This non-stationary property mainly concentrates on the edge part in the sinogram. If we neglected this fact, the edge information in the reconstructed image should be blurred.

In our Kalman filter design below, the non-stationary characteristic of the Poisson noise in the observation model was considered.

Kalman Filter

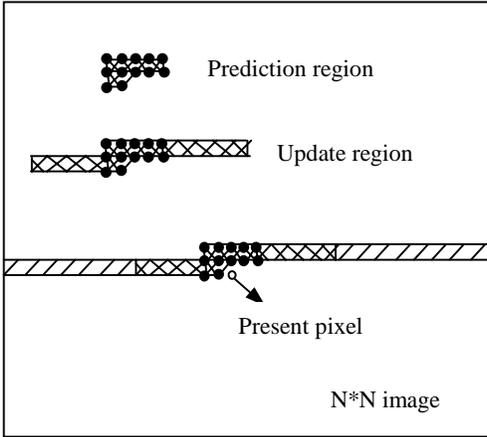


Figure 2. The diagrams of the NSHP model and reduced updated Kalman filter.

By Kalman filter approach, there is a very important question of how to determine both the signal and observation models. In this paper, the signal model of the Kalman filter is given by a non-symmetric half plane (NSHP) recursive model [6][7]. As can be seen in Figure 2 [6], the hollow circle (indicated by the arrow) represents the present pixel (or projection bin in our case); its intensity value can be predicted based on the intensity values of those black pixels, given present noisy observation. Through solving the Yule-Walker equations [6], we can obtain the coefficients of the NSHP model. In this paper, the signal model is given by equation (1):

$$s(m,n) = Fs(m-1,n) + w(m,n) \quad (1)$$

$$F = \begin{bmatrix} a_0 a_1 \dots a_{p \times N + p - 1} \\ 100 \dots \dots \dots 0 \\ 010 \dots \dots \dots 0 \\ \vdots \vdots \vdots \vdots \vdots \vdots \vdots \vdots \\ 000 \dots \dots \dots 1 \quad 0 \end{bmatrix}$$

where F is the system augmentation matrix and $w(m,n)$ is a white Gaussian noise. The value of the first row in the matrix F is given by the coefficients of the NSHP model. The size of the matrix F depends on the NSHP model order and the data size. Need to say, there are so many zeros in the first

raw. The power of the excitation noise is determined by the above Yule-Walker equation with the defined matrix F . Because it is impossible to have the noise free data to estimate the signal model coefficients, instead we utilize smoothed data, filtered by a 2-D Hanning low-pass filter on the sinogram after performing the Anscombe transformation, to calculate the signal model coefficients. In addition, the NSHP model of a 2x2 order was adopted in this paper.

For a linear imaging system as established by many researchers, the observation model is regarded to be the sum of the true signal and the white Gaussian noise. In our case, because it is a Poisson stochastic process for the SPECT sinogram, which is not additive noise, we cannot apply the additive model to describe the Poisson process. However, by applying the Anscombe transformation, Poisson noise can be transferred to Gaussian noise whose variance is shown in Figure 1. This transformation results in the expression of equation 2. This means that our Kalman filter has an ability to trace the non-stationary variation of the noise model in the Anscombe transformed space.

$$x(m,n) = Hs(m,n) + v(m,n) \quad (2)$$

$$H = [1,0,0,\dots,0].$$

According to the above two models, we can immediately write down the Kalman filter equations as:

Prediction:

$$P_b = FP_a F^T + Q_w \quad (3)$$

$$s_b = F \hat{s}_a. \quad (4)$$

Update:

$$K = P_b H (H P_b H^T + Q_v)^{-1} \quad (5)$$

$$\hat{s}_b = \hat{s}_a + K(x(m,n) - H \hat{s}_a) \quad (6)$$

$$P_a = (I - KH) P_b. \quad (7)$$

Finally, the inverse Anscombe transformation is applied to the data processed by Kalman filter above to obtain the final results, as can be seen by equation (8) below:

$$y = s^2 - 0.125 \quad (8)$$

where y is the final result and s is the data processed by the Kalman filter.

The computing load of above iterative process is quite heavy. In order to accelerate the computation, we adopt the reduced update strategy introduced first by J.W. Woods [6][7], see Figure 2. During the update phase, just those pixels within a certain distance from the present pixel are updated. This strategy saves significantly the computing time, without sacrifice of the accuracy.

In summary, our strategy of noise reduction consists of two steps: application of the Anscombe transformation and

use of the Kalman filter for accurate consideration of the non-stationary noise model.

Simulation

We utilized the digital MCAT phantom to evaluate our Kalman filter method and compared the results with that of other noise suppression methods. The relative activity levels of the heart, lungs, liver, kidneys, spleen, and sternum were 1.0, 0.03, 0.69, 0.84, 0.96, and 0.12, respectively, simulating the distribution of Tc^{99m} . Both parallel and fan beam collimators were selected. The influence of the scatter and attenuation was not included in this work. In this paper, projections of 128x128 array size at 128 views evenly spanning over 360° were simulated by the SIMIND MC program. The total photon counts are about 100k, which means that the projection data has a very high noise level.

III. RESULTS

Comparison of Different Noise Reduction Methods

First, we used a simple mathematic phantom (a double-ellipse phantom, which was used in several publications) that just has two ellipse objects to validate our Kalman filter. We applied the SIMIND MC program to generate the sinogram of this phantom. The Ramp filter, Hanning filter, 3-D Wiener filter and our Kalman filter were employed to treat the noisy sinogram. Then the FBP (filtered backprojection) algorithm was straightly applied to reconstruct the smoothed projection data.

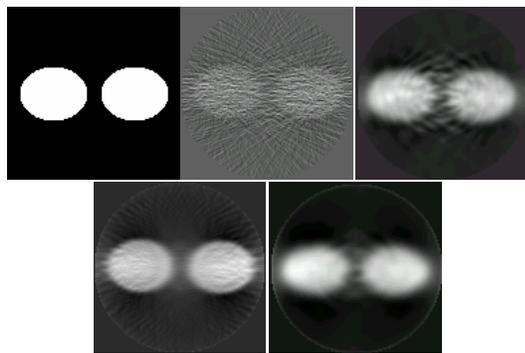


Figure 3. Comparison of double ellipse mathematic phantom results using different filtering methods. The images from left to right of the top row are: noise-free, Ramp filter, Hanning filter (cutoff frequency equals to 0.25), respectively. On the bottom row from left to right are: 3-D Wiener filter and our Kalman filter, respectively.

Figure 3 shows the noise free image and reconstructed images based on the different filtering methods. As can be seen in Figure 3, the result of the Ramp filter (without noise reduction) is very noisy in which we could hardly see any details. Although Hanning filter improves the quality of the reconstructed image, there are also a lot of artifacts. The result of the 3-D Wiener filter is better than the conventional

linear filter. Among all the methods, our Kalman filter generates the best result.

Figure 4 shows another example of the comparison study. The reconstructed images of the transverse MCAT slices by different noise suppression methods on the noisy projections are compared. Here, the noise free image and reconstructed images of the slice 31, 34, and 43 are shown. As seen in Figure 4, the results of the Ramp filter show that the noise level is very high. Although the conventional linear filter (Hanning filter) can reduce the noise level to a certain extent, there is a compromise for resolution and contrast. The Wiener filter can achieve a better tradeoff between the noise reduction and resolution preservation. Our Kalman filter approach for the non-stationary noise model shows a significant improvement by visual judgment, which not only suppresses the noise to the great extent, but also preserves the edge of the object very well.

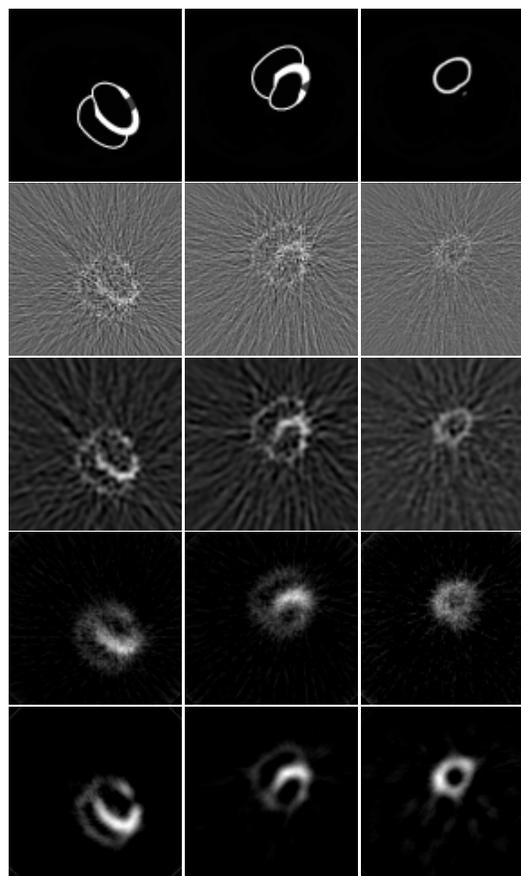


Figure 4. Comparison of reconstructed transverse MCAT slice images using different filtering approaches. The images from left to right represent: slice 31, 34, and 43, respectively. From top to bottom: noise-free, Ramp filter, Hanning filter (cutoff frequency equals to 0.25), 3-D Wiener filter and our Kalman filter, respectively.

It is known that the frequency spectrum of the noise in the projection data not only presents in the high frequency region, but also in the low frequency region. When the linear low-pass filter eliminates the noise in the high frequency region, it also eliminates the useful information of the true signal there. Moreover, it does not suppress the noise in the low frequency region. Therefore, it is not effective. At this point, Wiener filter is obviously a better choice than the linear low-pass filter. However, Wiener filter is based on the stationary random process. For the SPECT sinogram, this prerequisite is not exactly tenable. The variance of the noise at the boundary region in the sinogram is non-stationary, which leads to poor edge property of the reconstructed image with the Wiener filter if the variance is regarded as a constant. Therefore, the Kalman filter for the non-stationary noise model is obviously a better choice than other methods. In this studied case, the total photon counts were just 100k, which means that it just took a little scan time or very low radiotracer concentration. Even though in such higher noise environment, the Kalman filter can obtain good results after the Anscombe transformation.

Comparison of Different NSHP Model Orders

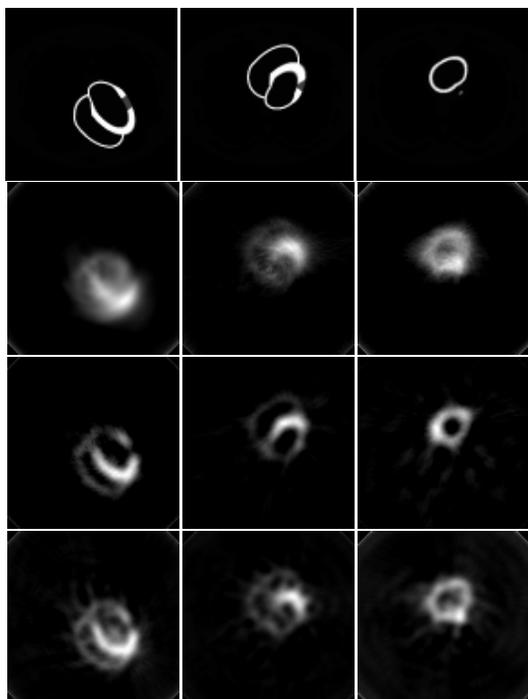


Figure 5. The influence of a different setting of the NSHP model order on the reconstruction results, based on MCAT phantom study. The images from left to right represent: slice 31, 34, and 43, respectively. From top to bottom are: noise-free, Kalman filter corresponding to 1×1 , 2×2 , and 3×3 NSHP model orders, respectively.

Figure 5 shows the influence of different NSHP model orders on the filtered results. As can be seen in this figure, the 2×2 NSHP model order can obtain the best result in

comparison with other settings. The 1×1 NSHP model order is too small to get accurate estimation of the signal model, so it is expected that its result is not so satisfied. Theoretically, 3×3 NSHP model order could be better than 2×2 . In some literatures [6][7], the NSHP model order was mostly set up as 3×3 or more. In order to investigate the influence of the NSHP model order on the Kalman filter, the double ellipse mathematic phantom above and the MCAT phantom with the parallel collimator and fan beam collimator were adopted. Table 1 shows the acceptable degree of the reconstructed images of different phantoms based on different NSHP model orders.

Table 1. Acceptable degree of the reconstructed images of different phantoms based on different NSHP model orders.

Model order \ Phantom	1×1	2×2	3×3
Double ellipse	NO	YES	YES
MCAT (parallel)	NO	YES	NO
MCAT (fan)	NO	NO	YES

For the double ellipse phantom, a 2×2 or higher model order can obtain satisfactory results. In contrast with the MCAT phantom with parallel beam collimator, for the MCAT phantom with fan beam collimator, the 2×2 NSHP model order is not enough to get accurate estimation of the signal model. The 3×3 NSHP model order is necessary to get reasonable results for this phantom. For parallel beam collimator, the 3×3 NSHP model order is acceptable, although it is slightly worse than 2×2 NSHP model order. It is clearly known that there is not a NSHP model order, which is appropriate for any situation. These simulation results demonstrated that the Kalman filter is sensitive to the NSHP model order for the SPECT sinogram.

IV. CONCLUSIONS

In this paper, we proposed a new noise reduction method for non-stationary noise model of SPECT sinogram based on Kalman filter. Compared to other noise reduction strategies, the simulation experiment demonstrated that the results of our Kalman filter method show obvious improvement by visual judgment. The Kalman filter is based on the non-stationary noise model and can obtain excellent results even under the condition of a very high level noise. Furthermore, we demonstrated that Kalman filter is sensitive to the setting of NSHP model order. The model order must be carefully selected for a specific application.

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REFERENCES

- [1] F.J. Anscombe, "The transformation of Poisson, binomial and negative-binomial data," *Biometrics*, **35**: 246-254, 1948.
- [2] J.H. Cheng, Z. Liang, J. Ye, and et al, "Inclusion of *a priori* information in frequency space for quantitative SPECT imaging," Conference Record of IEEE NSS-MIC, 1996.
- [3] P.P. Hannequin, J.F. Mas, "Application of iterative and non-stationary smoothing filters for statistical noise reduction in nuclear medicine," *Nuclear Medicine Communications*, **19** (9): 875-885, 1998.
- [4] H. Lu, G. Han, D. Chen, et al. "A theoretically based approach to noise reduction in SPECT by Anscombe and K-L transforms," *J. Nucl. Med.* **41**(5): 893, 2000.
- [5] J. Kim and J.W.Woods, "Spatio-temporal adaptive 3-D Kalman filter for video," *IEEE Trans Image Processing*. **6**: 414-424, 1997.
- [6] J.W.Woods and C.H. Radewan, "Kalman filtering in two-dimensions," *IEEE Trans. Inform. Theory*, **23**: 473-482, 1977.
- [7] S.M.Kay. *Modern spectral estimation*. Prentice Hall. pp 492 - 504, 1988.