Noise Reduction for Four Dimensional Dynamic Computed Tomography

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***Abstract*--Four-dimensional computed tomography (4D-CT) plays an important role in treatment planning for radiation therapy. By incorporating temporal information in CT scan, the motion of tumor can be modeled and the accuracy of dose delivery can be improved in radiation therapy. To acquire temporal information, repeated scan is required at the same slice position during respiratory cycles. Therefore, the X-ray radiation delivered to patients during 4D-CT procedure is much higher than traditional 3D-CT. One simple and cost-effective way to reduce the X-ray radiation is to acquire CT data with low mAs. However, the image quality will degrade in low mAs CT images due to high noise and noise-induced artifacts. In this work, we utilize the Karhunen-Loève (KL) transform to consider the dynamic nature and apply a penalized weighted least-squares (PWLS) method to adaptively treat the non-stationary noise among all the 3D KL principal components, followed by filtered back-projection on the principal components for 4D image reconstruction. A feasibility study on this KL domain PWLS low-dose 4D-CT approach was performed by a patient study with very encouraging outcomes. Evaluation on a large sample size is under progress.**

I. Introduction

In radiation therapy, accurate dose delivery to the target volume is critical. In the lung and upper abdominal regions, the target volume is moving due to respiration. Four-dimensional computed tomography (4D-CT) with respiratory gating provides dynamic volume imaging datasets of the moving organs. From the 4D dynamic datasets, the trajectory of the target can be modeled. Therefore, precise location of the target tumor during treatment planning can be traced and accurate dose delivery can be achieved as patient breath freely during treatment [1-4].

Manuscript submitted by January 30, 2007. This work was supported in part by the U.S. NIH under Grant No. CA082402 and CA110186. Dr. H. Lu was supported by the National Nature Science Foundation of China under Grant 30470490.

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Compared with conventional 3D-CT, the dynamic 4D-CT provides information in the time domain at each location. To obtain temporal information, repeated measurements are acquired at the same location. The exposure of a 4D-CT will be much higher than a single 3D-CT. For example, a 4D-CT dataset usually consists of ten 3D-CT datasets at different phases of a breathing cycle. Thus the radiation exposure during the data acquisition is at least 10 times higher than the conventional 3D-CT with the assumption that the protocols are kept the same during patient scanning. This will result in unacceptable radiation to the patient. Lower the radiation exposure will sacrifice the image quality due to excessive noise and noise-induced artifacts, if the same image reconstruction algorithm is used. Effective noise treatment on the dynamic data sequence of low mAs protocols is necessary to suppress the noise and remove the noise-induced artifacts.

Karhunen-Loève (KL) transform (also known as principal component analysis) has been recognized as a useful tool to process dynamic data sequence [5-7] and specifically to treat a non-stationary noise adaptively in a correlated data sequence with [8, 9]. In this work, we adapted our KL domain penalized weighted least-squares (PWLS) method for sinogram noise reduction from 3D spatial space to 4D spatio-temporal domain for low-dose 4D-CT application [10]. The PWLS criterion was used to model the first and second moments of log-transformed low mAs sinogram, where the noise model of the log-transformed sinogram was established from repeated phantom measurements [11, 12]. The KL transform plays two roles in the method: (i) reduce the 4D problem to a 3D problem, i.e., the complicated 4D prior [13] now is reduced to a 3D spatial prior after the KL transform and, therefore, simplify the minimization of the associated objective function; and (ii) de-correlate the signal correlation among different phases of the dynamic data sequence and facilitate adaptive noise treatment among their corresponding independent principal components.

II. Method

A. KL domain PWLS for 4D-CT sinogram restoration

We first employ the KL transform to account for the correlation among different phases of the dynamic CT sinogram series. Three neighboring phases of 3D-CT sinograms were chosen to perform the KL transform. The KL transform is defined as:

$$\tilde{y} = A\,\hat{y} \tag{1}$$

where $\hat{y}$ is the matrix of selected neighboring phases sinogram data, $\tilde{y}$ is the KL transformed $\hat{y}$, and A is the KL transform matrix which is calculated based on:

$$K_t A' = A' D. \quad (2)$$

In equation (2), K_t is the covariance matrix of the selected neighboring phases sinogram data and $D = diag\{d_l\}_{l=1}^{2n+1}$, with d_l being the *l*-th eigenvalue of K_t. It is noted that the covariance matrix of the KL transformed data signal is diagonal, which means that the covariance of the signal between different phases after the KL transform will be zero. Therefore, the data signals of different KL components are no longer correlated and each KL principal component can be processed separately.

After the KL transform, we utilized the PWLS criterion to restore each KL component. The PWLS criterion is equivalent to the maximum *a posterior* (MAP) of independent Gaussian distributed noise. Indeed, the sinogram noise after the log-transform and system calibration can be well approximated by a Gaussian distribution from both the analysis on repeated phantom measurements [11, 12] and the theoretical derivation based on the Poisson noise model of the detected signal (i.e., the raw data prior to the logarithmic transformation) [14]. Most importantly the relationship between the variance and the mean was determined, which gives an accurate weight for the PWLS criterion. In the KL domain, the PWLS criterion is expressed as:

$$\Phi_l(\tilde{q}_l) = (\tilde{y}_l - \tilde{q}_l)' \tilde{\Sigma}_l^{-1} (\tilde{y}_l - \tilde{q}_l) + (\beta / d_l) \tilde{R}(\tilde{q}_l) \quad (3)$$

where $\tilde{q}_l$ is the *l*-th KL components of q and q is the ideal sinogram to be estimated. The first term in equation (3) is the WLS criterion and the second term is the penalty term which reflects the prior information of the noise-free sinogram. In this work, a quadratic form penalty was used:

$$\tilde{R}(\tilde{q}_l) = \tilde{q}_l' \tilde{R} \tilde{q}_l = \tfrac{1}{2} \sum_i \sum_{m \in N_i} w_{im} (\tilde{q}_{i,l} - \tilde{q}_{m,l})^2 \quad (4)$$

where N_i indicates the set of six nearest (or first-order) neighbors of the *i*-th voxel in each of the 3D KL component. The parameter w_{im} was defined as equal to 1 for the two horizontal neighbors along the bin direction, 0.5 for the two vertical neighbors along the view angular direction, and 0.25 for the vertical neighbors along the axial direction. Different values of w_{im} along different dimension basically reflect anisotropic coupling strength along bin, view and axial direction in the sinogram space.

In equation (3), β is the penalty parameter which controls the trade-off between the measured data and the prior knowledge of the sinogram. A small β reflects weak coupling among neighboring voxels and the solution of minimizing equation (3) is noisy. It is worth to point out that in the KL domain the penalty parameter become β / d_l, which is adaptive to the eigenvalue of the KL component. The KL component with a smaller eigenvalue is usually associated with a lower signal to noise ratio (SNR). Therefore, a larger penalty should be applied on this KL component with lower SNR, an attractive property of self adaptiveness.

In addition to the penalty term in the PWLS objective function, another important term is the weight. The weight determines the performance of the PWLS algorithm. The weight is usually determined by the variance of corresponding data. The data with a large variance reflects that the measurement is less reliable and, therefore, it should contribute less to the objective function. In this study, the variance of the sinogram data can be calculated from an analytical formula [11, 12], which describes the relationship between the mean and the variance of the low mAs sinograms.

Minimization of the objective function (3) can be calculated efficiently via the Gaussian-Seidel updating strategy [14]. The iterative formula for minimizing equation (3) is given by:

$$\tilde{q}_i^{(n+1)} = \frac{\tilde{y}_i + \dfrac{\beta}{d_l} \tilde{\sigma}_i^2 \left(\sum_{m \in N_i^-} w_{im} \tilde{q}_m^{(n+1)} + \sum_{m \in N_i^+} w_{im} \tilde{q}_m^{(n)} \right)}{1 + \dfrac{\beta}{d_l} \tilde{\sigma}_i^2 \sum_{m \in N_i} w_{im}} \quad (5)$$

in which index *n* is the iterative number, N_i^- denotes the three nearest neighbors of voxel *i* whose index number are smaller than *i*, N_i^+ denotes the three nearest neighbors of voxel *i* whose index number are larger than *i*, and N_i denotes these six nearest neighbors of voxel *i*.

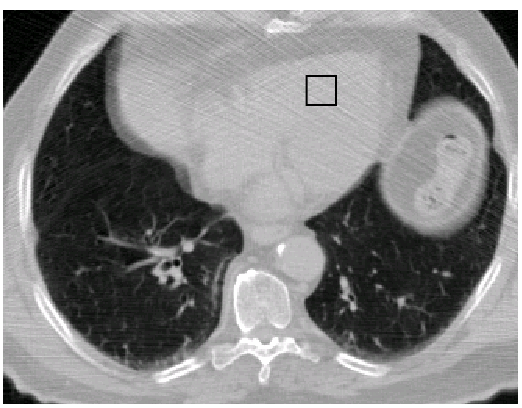
After the inverse KL transform on the PWLS-processed KL components, we obtain the restored sonogram series. The 4D-CT image was then reconstructed by filtered back-projection (FBP) from the KL-PWLS restored 3D sinogram sequence.

B. *Data Acquisition*

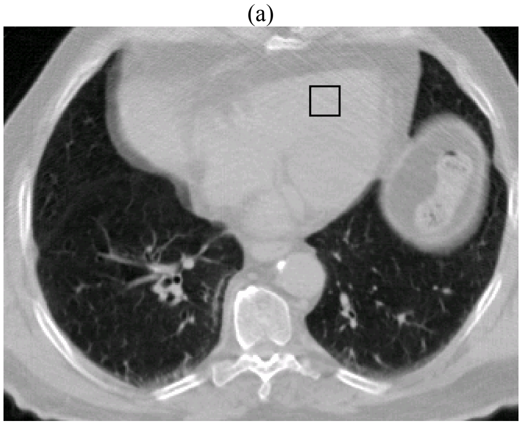
In this work, we tested the effectiveness of the KL-PWLS sinogram restoration method using a patient study. The clinical 4D-CT patient study was performed with a Discovery ST LightSpeed 8-slice PET/CT scanner (GE Medical Systems). The patient was asked to breathe normally during the scan. The plastic block with two infrared reflective markers was taped on the top of the patient's abdomen, placed medially and a few cm inferior to the xiphoid processes. The respiratory signal of the patient from the real-time position management (RPM) system (Varian Medical Systems, Palo Alto, CA) was recorded and synchronized with the CT data acquisition. The axial coverage of the patient was 25 cm. Other scan parameters were 130 mAs, 120 kV, and 2.5 mm slice thickness. The cine interval between images was at 0.45 seconds. Each image reconstruction used 360° of data corresponding to 0.8 seconds duration. After the scan data were prospectively reconstructed at the PET/CT scanner, both the CT images and the corresponding motion data recorded by the RPM system were transferred to a GE Advantage Workstation (GE Medical Systems, Waukesha, WI). The “Advantage 4D” software on the workstation simultaneously displays the CT images and the motion data, and sorts the cine images into a set of respiratory phase images. In this study, a total of 10 phases were created, with phase intervals of 10% of the respiratory cycle. Since the CT system does not allow the readout of the sinogram series, we simulated the dynamic data series by re-projecting the image sequence using the CT system’s X-ray source and detector configuration.

III. Results

The re-projected sinogram series were first reconstructed by a standard FBP (i.e., with the Ramp filter at 100% Nyquist cutoff) frame-by-frame and the results are identical, as expected, to the original images from the CT system. Figure 1(a) shows an example of an image slice. The same sinogram series were then processed by the KL-PWLS noise reduction method, followed by the same standard FBP reconstruction. Figure 1(b) shows the result of the KL-PWLS at the same slice location of Figure 1(a). It is clearly observed that the streaking artifacts are efficiently suppressed. The detail preservation of the KL-PWLS method can be observed when the same images are displayed in a different window level, see Figure 2. Figure 3 displays the vertical profiles of the two images in Figure 1 at row 181, which concurs with the results of Figure 1 and Figure 2, i.e., suppression of noise with preservation of important structures in the image slice.

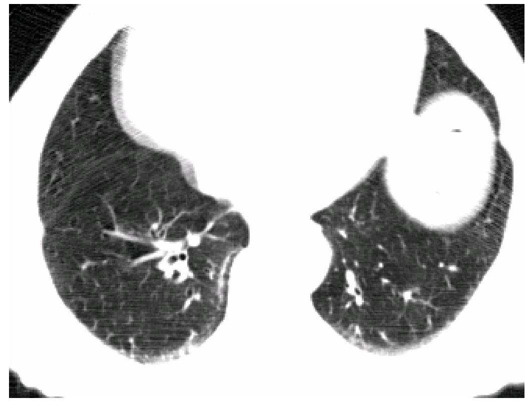


(a)



(b)

Figure 1: Test result of the KL-PWLS sinogram restoration for low-dose 4D-CT. (a)-the original image and (b)-the KL-PWLS processed image The display window is [0 1500].



(a)

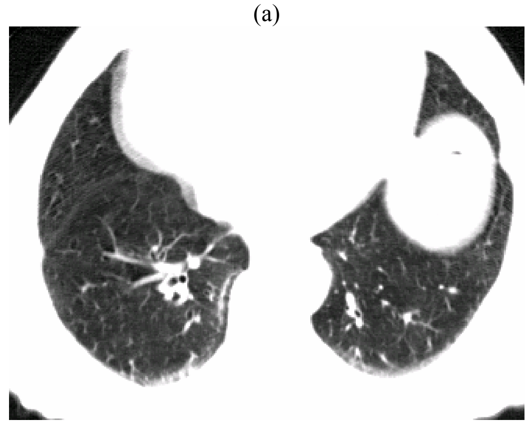


Figure 2: The images of Figure 1 with display window [0 600].

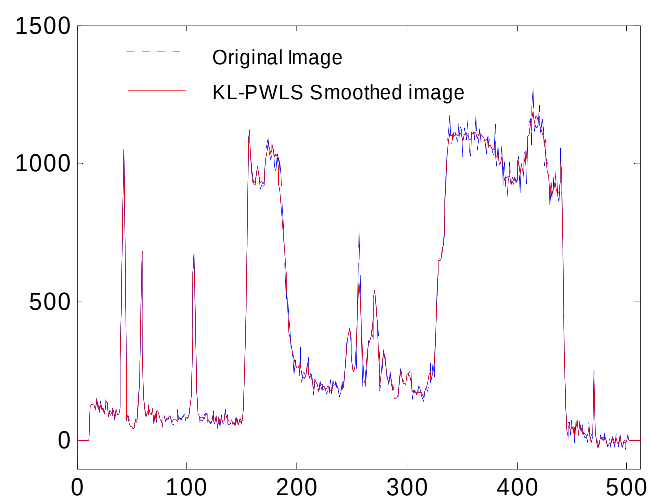


Figure 3: Vertical profiles of the images in Figure 1 at row 181.

To more quantitatively show the performance of the KL-PWLS method, we calculated the SNR of a region of interest (ROI) from the images in Figure 1. The results are listed in Table 1. It is seen that the SNR of the selected ROI for the image after the KL-PWLS noise reduction was more than 3 times higher than that of the original image from the CT scanner.

Table 1: Mean, standard deviation and SNR of selected ROI in Fig. 1.

	Mean value (HU)	Standard deviation (HU)	Signal to Noise Ratio
Original	1097.2	38.4	28.5
KL+PWLS	1095.2	11.0	99.6

IV. DISCUSSIONS AND CONCLUSIONS

In this work, we adapted our previous KL domain PWLS algorithm from 3D spatial space to 4D spatio-temporal domain for noise reduction on the sinogram series. The KL transform was performed among the neighboring phase of the sinogram series to de-correlate their correlations and the PWLS criterion was applied adaptively to restore each independent KL principal component. In the 4D sinogram dataset, there is a strong correlation among different phases of the 3D dataset series. One conventional method to deal with this kind of correlation is to design a 4D prior in an iterative image reconstruction [13]. A 4D prior is difficult to design due to the irregular movement of tumors and organs. In fact, a great effort has been devoted to model the motion of tumors during radiotherapy and a great work remains to be done for the motion modeling, see a recent review by Webbs [15]. The KL analysis provides an excellent alternative means to handle the correlation among different phases of the 3D dataset series for noise reduction, and the adaptive PWLS noise treatment on the de-correlated KL principal components showed encouraging result by the presented patient study.

Li *et al.* [4] has proposed a 4D-PWLS smoothing method in the image domain. Compare with Li's method [4], our sinogram domain KL-PWLS algorithm at least has three advantages. (1) The sinogram domain method is based on statistical noise modeling of the measured data, where the weight in the PWLS is determined accurately from an explicit formula that describes the relationship between the mean and the variance of the measured sinogram data. In [4], the noise modeling of the reconstructed images is not clear and the weight was estimated using local image variance. (2) By the KL transform to consider the correlation among different phases, one dimensional freedom (i.e., penalty parameters) in the prior model is eliminated (i.e., 4D prior is reduced to 3D). (3) By the KL transform, the registration step used in [4] is avoided since the KL transform automatically consider the difference (as well as similarities) between neighboring phases of the 4D data sequence. However, a thorough study to compare the sinogram domain KL-PWLS method with the image domain 4D-PWLS is desired and worth for further investigation effort.

V. REFERENCES

[1] D.A. Low, M. Nystrom, E. Kalinin, P. Parikh, J.F. Dempsey, J.D. Bradley, S. Mutic, S.H. Wahab, T. Islam, G. Christensen, D.G. Politte, and B.R. Whiting, "A method for the reconstruction of 4-dimensional synchronized CT scans acquired during free breathing", *Medical Physics*, Vol. 30, pp. 1254-1263, 2003.

[2] T. Pan, T. Lee, E. Rietzel, and G.T.Y. Chen, "4D-CT imaging of a volume influenced by respiratory motion on multi-slice CT", *Medical Physics*, Vol. 31, pp. 333-340, 2004.

[3] W. Lu, P.J. Parikh, J.D. Bradley, and D.A. Low, "A comparison between amplitude sorting and phase-angle sorting using external respiratory measurement for 4D CT", *Medical Physics*, Vol. 33, pp. 2964-2974, 2005.

[4] T. Li, E. Schreibmann, B. Thorndyke, G. Tillman, A. Boyer, A. Koong, K. Goodman, and L. Xing, "Dose reduction in 4D computed tomography", *Medical Physics*, Vol. 32, pp. 3650-3659, 2005.

[5] D.C. Barber, "The use of principal components in the quantitative analysis of gamma camera dynamic studies," Phys. Med. Biology, Vol. 25, pp. 283-292, 1980.

[6] M.N. Wernick, E.J. Infusino, and M. Milošević, "Fast spatio-temporal image reconstruction for dynamic PET", *IEEE Trans. Medical Imaging*, Vol. 18, pp. 185-195, 1999.

[7] M.V. Narayanan, M.A. King, M.N. Wernick, C.L. Byrne, E.J. Soares, and P.H. Pretorius, "Improved image quality and computation reduction in 4-D reconstruction of cardiac-gated SPECT images", *IEEE Trans Medical Imaging*, Vol. 19, pp. 423-433, 2000.

[8] H. Lu, G. Han, D. Chen, L. Li, and Z. Liang, "A theoretically based pre-reconstructing filter for spatio-temporal noise reduction in gated cardiac SPECT imaging", *Conf Record IEEE NSS-MIC*, in CD-ROM, 2000.

[9] H. Lu and Z. Liang, "Spatial resolution characteristics of KL domain adaptive Wiener filtering on SPECT sinogram Poisson noise", *International Conference on Fully 3D Reconstruction in Radiology and Nuclear Medicine*, 2005.

[10] J. Wang, T. Li, H. Lu, and Z. Liang, "Penalized weighted least-squares approach to sinogram noise reduction and image reconstruction for low-dose X-ray computed tomography", *IEEE Trans. Medical Imaging*, Vol. 25, pp. 1272-1283, 2006.

[11] H. Lu, I. Hsiao, X. Li, and Z. Liang, "Noise properties of low-dose CT projections and noise treatment by scale transformations", *Conf Record IEEE NSS-MIC*, in CD-ROM, 2001.

[12] T. Li, X. Li, J. Wang, J. Wen, H. Lu, J. Hsieh, and Z. Liang, "Nonlinear sinogram smoothing for low-dose X-ray CT", *IEEE Trans. Nuclear Science*, Vol. 51, pp. 2505-2513, 2004.

[13] D.S. Lalush and B.M.W. Tsui, "Space-time Gibbs priors applied to gated SPECT myocardial perfusion," *International Conference on Fully 3D Reconstruction in Radiology and Nuclear Medicine*, 1995.

[14] K. Sauer and C. Bouman, "A local update strategy for iterative reconstruction form projections". *IEEE Trans. Signal Processing*, Vol. 41, pp. 534-548, 1993.

[15] S. Webb, "Motion effects in (intensity modulated) radiation therapy: a review", *Phys. Med. Biology*, Vol. 51, pp. 403-425, 2006.